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Ophthalmic Technologies XV

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22 January 2005
San Jose, USA

Sponsored by
U.S. Air Force Office of Scientific Research
SPIE – The International Society for Optical Engineering

Proceedings of SPIE
Volume 5688
The dependence of the apparent source on exposure position

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ABSTRACT

For a given power entering the eye, the level of retinal thermal hazard depends on the retinal image size over which that power is distributed. Maximum permissible exposure limits are given in terms of the angular subtense of the apparent source, which describes the diameter of the retinal image. Based on a simple beam propagation model for a laser beam being transformed by the eye, it will be discussed that both the location as well as the angular subtense of the apparent source depend on the exposure position of the eye in the beam. For a given position, it is important to consider different accommodation conditions of the eye to determine the location and angular subtense of the apparent source. Only when the eye is fixed in the relaxed condition is the angular subtense of the apparent source equivalent with the far field divergence for any exposure position. For a Gaussian beam, when the eye is located in the far field, the beam waist can be considered as the apparent source, while when the eye is located at or close to the beam waist, the apparent source is located in infinity and the angular subtense of the apparent source becomes equivalent with the far field beam divergence.

Keywords: laser safety, hazard analysis, product classification, apparent source, maximum permissible exposure, allowable emission limit, IEC 60825-1

1. INTRODUCTION

For the quantification of the potential hazard of a given source of optical radiation to induce retinal injuries, it is important to determine the power that passes through a measurement aperture, to simulate the radiation level that would enter the eye. However, for retinal damage, it is clear that not only the power that enters the eye is the critical figure, it is also important to consider over which area on the retina that power is distributed: if the power, as is the case for well collimated laser beams, is focused to a minimal spot (with a nominal angular subtense of 1.5 mrad), then already low power levels (less than 1 mW in the extreme) can produce retinal damages. If the power that enters the eye is spread over a larger area of the retina, the allowed power level can be hugely increased, for instance by up to roughly a factor of 500 000 if the radiation is spread over the whole fundus. The diameter of the minimal image at the retina is characterised by the parameter \( \alpha \), which is referred to as the “angular subtense of the apparent source” (see also figure 1).

Figure 1. The power that enters the eye and the area over which it is spread on the retina are the two important factors to assess for a safety of optical radiation that can damage the retina.

The dependence of the thermal hazard exposure limit, if they are given in terms of power into the eye, depends linearly on the diameter of the retinal image (up to angle of 100 mrad). I.e. between the minimal image (1.5 mrad) exposure limit and the limit for an image of 100 mrad, for the larger image, the power that enters the eye can be a factor 66 higher. Beyond 100 mrad, the dependence on \( \alpha \) is squared when the source is homogeneous. Laser exposure limits (and product emission limits) are basically given in terms of “power into the eye”, or more exact, power through an aperture

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(exposure limits are often given in terms of irradiance at the cornea, but averaged over an aperture with 7 mm diameter, which can be transformed into a ‘power through an aperture’ value by multiplication with the area of the averaging pupil). As the retinal irradiance (or more generally, the irradiance in the image plane) is directly proportional to radiance, broadband exposure limits (where extended sources are the default case) are specified as radiance values. For the case of radiance limits, for the thermal retinal limit, the limit decreases with increasing retinal image size. This is based on the better cooling of small retinal spots, where for the same retinal irradiance level in the image, a smaller spot can take higher irradiance levels than a bigger one where the radial cooling is less efficient (for a more detailed discussion on exposure limits and safety related radiometry the interested reader is referred to Henderson and Schulmeister 2004 [1]). By accounting for the measurement FOV (or when the source is homogenous, the source size), then radiance limits can be transformed into ‘power through an aperture’ limit, and vice versa.

1.1 Scheme for a complete safety analysis
For a complete safety analysis, any position of the eye within the beam and, for each position, any accommodation state of the eye should be considered. The range of accommodation in the IEC laser safety standard IEC 60825-1 is implied as to range from 10 cm to infinity, i.e. the near point of the eye is assumed as 10 cm. The discussion in the following concentrates on retinal thermal hazard, as the actual size of the source for the characterisation of the photochemical hazard only depends on retinal irradiance, and not on the diameter of the actual image.

The current edition of the international laser safety standard IEC 60825-1 specifies the measurement distance relative to the location of the apparent source, which introduces difficulties of interpretation when it is not clear where and what the apparent source is. In the following, a generally applicable safety analysis concept is proposed where the assessment of the exposure or emission level (for instance for classification of the source) and the determination of the angular subtense of the apparent source is done at the most hazardous position and not relative to the location of some abstract apparent source. The most hazardous position is where the ratio of power as determined with a 7 mm aperture over the angular subtense of the apparent source \(\alpha\) is at its maximum, as will be further elaborated below.

1.2 Power Trough Aperture
The power that passes through a given measurement aperture (for instance with a diameter of 7 mm) depends on the beam diameter at the position of measurement, as indicated in figure 2. While it is obvious that the highest power level is accessible at the source, this might not be the most hazardous position for retinal hazards, as the angular subtense of the apparent source may be large when the eye is at that position.

![Figure 2](image)

**Figure 2.** The power that passes through a given aperture (representative of the pupil of the eye) depends on the beam diameter, and therefore of the measurement position (eye position) in the beam.

1.3 Retinal image size
For a given source emitting optical radiation (a beam), and for a given exposure position of the eye in the beam, the retinal image depends on the accommodation state of the eye. For a safety analysis, any possible accommodation state needs to be considered and the worst case image, which in simple terms is the smallest image, needs to be used for the analysis of the retinal thermal hazard. For a given position of the eye in the beam, it is then this minimal image size which can be obtained by accommodation that is characterised by the angle \(\alpha\). It is only in animal studies that the state of accommodation is fixed when the animal is anaesthetized and the eye is relaxed, i.e. accommodated to infinity. Only in this special case is the angular subtense of the retinal image given by the far field divergence of the beam. For the general human hazard analysis, accommodation generally needs to be considered.
For a simple source without optical elements such as reflectors or lenses, for instance a clear light bulb or a frosted light bulb, it is obvious what the source of radiation is, and accommodation to that source will produce the highest retinal irradiance (a sharp image of the filament or of the diffusely transmitting light bulb, respectively). Equivalently, for a bare LED chip without any reflecting cup surrounding the chip nor any lens in front of the chip, the chip itself can be considered as the apparent source and the emitting area can be used to determine the angular subtense of the apparent source.

If optical elements are part of the source, the determination of the apparent source is less straightforward, as for instance for LEDs the light reflected by the cup represents a different source than the chip itself and the source is both magnified and often also distorted by a collimating (projecting) lens as part of the package of the LED. In that case, the analysis is best performed by imaging the source onto an array detector such as a CCD camera and simulating the accommodation of the eye (changing the focal length of the lens of the eye) by varying the distance between the imaging lens and the detector, as indicated in figure 4.

The aperture in front of the imaging lens has a diameter of 7 mm, as the size of the image may depend on the aperture and although the aperture for the determination of the angular subtense of the source is not specifically specified in the current safety guidelines and standards, this value follows from the requirement to use a 7 mm aperture for the determination of the power (and an even higher level of completeness of the radiation safety assessment would include varying the power and imaging aperture (in lieu) between 1 mm and 7 mm for the analysis to simulate different pupil sizes). The CCD array is moved between the focal plane of the lens (to image infinity) and the image position which is the conjugated plane of 10 cm in front of the lens, to simulate the near point accommodation of the eye. For each lens-array distance, the irradiance profile on the CCD is different and has an associated characteristic diameter, from which the angular subtense \( \alpha \) of that image is derived. The CCD position that represents the most hazardous value of \( \delta \) is determined (which is usually the smallest value) and this value is then referred to as the angular subtense of the apparent source \( \alpha \) (for the given position of the lens/eye in the beam), i.e. \( \alpha = \min \delta \). For the CCD position which produces the most hazardous image, the irradiance profile on the CCD is the image of the apparent source. This is in lieu of the definition of the angular subtense of the apparent source as the minimal image size that can be obtained for a given source (and for a given position of the eye in the beam).
1.4 The location of the apparent source
Once the most critical CCD array position is found (i.e. the one which produces the smallest, most hazardous image), the lens formula can be used to calculate the conjugated point in the object space, which is considered to be the location of the apparent source. This highlights the rather abstract character of the ‘apparent source’, which is the reason why it is referred to as ‘apparent’ source: it is independent of the actual physical source of radiation, it rather characterises the position to which eye has to accommodate to produce the smallest (most hazardous) retinal image. It should be noted that the location of the apparent source may well depend on the position of the eye in the beam, unless for simple sources without optics, where the physical source of radiation is also the apparent source. In the extreme, for a laser beam for instance, for exposure distances far enough of the beam waist (the Raleigh range), the apparent source is the beam waist, while for exposure positions at the beam waist, the apparent source moves back towards infinity.

1.5 The Most Hazardous Position
The basic principle of a hazard assessment or of a product classification is to assess the exposure level (or for product classification of the emission level) and compare it with an exposure limit (or for classification against some emission limit). Since the retinal thermal hazard limit when expressed as power through an aperture depends on $\alpha_i$ (i.e. the larger $\alpha_i$ the smaller the hazard), a relative thermal hazard level (RTH) can be defined which is proportional to $P / \alpha_i$, where $P$ is the exposure or emission level determined according to some rules (for instance the power that passes through an aperture with a diameter of 7 mm). Since both the power $P_i$ and the angular subtense of the apparent source $\alpha_i$ depend (in a different way) on the position of the eye in the beam (see figure 5), the retinal hazard level also depends on the position of the eye in the beam:

$$RTH_i = \frac{P_i}{\alpha_i}$$

The most hazardous position of the eye in the beam is simply the position where the $RTH_i$ assumes its maximum value.

![Figure 5. The generally applicable hazard evaluation method is based varying the position of the 'eye' within the beam and determining the position with the maximum value of the retinal thermal hazard value.](image)

For laser beams it is possible to set up a model based on beam propagation through lenses to determine for a given beam (characterised by the beam waist diameter and by the far field divergence) the most hazardous position (MHP), the angular subtense of the apparent source at that position and the power that enters eye at the MHP (see section 3). However, it is possible to determine the most hazardous position experimentally, and two example measurement setups and irradiance profiles are shown in the following section.

Figure 6 shows the schematic of examples for a measurement set-up and obtained images for one given position of the eye in the beam (7.5 cm from the tip of the LED) but for two different states of accommodation, in the top figure for the assumed near point of the eye, i.e. 10 cm which for the chosen distance of the lens to the LED happens to image the chip, which is magnified and optically recessed by the lens package of the LED, so that the optical conjugate to the image of the chip is located 2.5 cm behind the tip of the LED. In the lower example, the CCD array is located at the focal plane of the lens, and therefore images infinity, i.e. simulates a relaxed eye. The comparison of the retinal thermal hazard level (arbitrary units, for comparison between images only) shows that the top one represents a higher hazard level.
2. BEAM PROPAGATION TO UNDERSTAND ‘THE APPARENT SOURCE’ FOR LASER BEAMS

It follows from principles of beam propagation of a Gaussian beam [2], that the location of the apparent source is the centre of curvature of the wavefront that is incident on the eye (see figure 7). That is, when the eye accommodates to the position of the ‘origin’ of the wavefront that is incident on the eye, the smallest retinal spot size (for the given position of the eye in the beam) is created. The eye images the irradiance profile at this position and thus the beam diameter at the position of the centre of curvature of the wavefront also can be used to calculate the angular subtense of the apparent source.

In the far field, i.e. sufficiently far outside of the Raleigh range, the wavefront is close to a spherical one which originated at the location of the beam waist, so that for this condition, the beam waist position is a good approximation for the location of the apparent source (Figure 8 a). Therefore, for positions of the eye in the far-field, the angle that is subtended by the beam waist is the angular subtense of the apparent source $\alpha$. For positions closer to the beam waist, the wavefront becomes flatter and the “apparent source” of the wavefront that is incident on the eye moves away from the location of the beam waist. For an eye located in the beam waist (Figure 8 b), the smallest retinal spot size is obtained when the eye is relaxed and images the beam profile at infinity, which means that $\alpha$ equals the beam divergence (corrected for the different diameter definitions for the beam divergence and $\alpha$). It is only in this case that...
the angular subtense of the apparent source is equal to the beam divergence – for the general case, i.e. for exposure outside of the region of the beam waist, the angular subtense of the apparent source $\alpha$ is smaller than the divergence!

\[ \alpha = \frac{d}{z} \]

Figure 8 a. When the eye is located in the far field, the wavefront incident on the eye is spherical with the origin at the beam waist. Thus, in this case, the beam waist can be considered as the ‘apparent source’ both in terms of location and size. Figure 8 b) When the eye is located in the beam waist (or close to the beam waist), then the retinal smallest image is obtained when the eye accommodates to infinity – which is also the origin of the plane wavefront, that is incident on the eye when the eye is at the position of the beam waist.

This shows that there is no such thing as ‘the’ apparent source for a given beam, as the location (and size) of the apparent source depends on the position of the eye in the beam. Also the term ‘angular subtense of the apparent source’ is so abstract for the case of a laser beam that a better term would be to refer to the angular subtense of the minimal retinal image rather than of some ‘apparent source’. The term apparent is also not fully correct, as in some cases, the minimal (most hazardous) image is produced for accommodation states of the eye which for most people would not occur naturally. I.e. when looking into a source to determine the ‘apparent’ source visually in terms what the eye perceives, although the eye has the tendency to look for the sharpest image, this is in some cases not the most hazardous one and therefore should not be mistaken to be always characteristic of the “angular subtense of the apparent source”. An example are line lasers where the eye accommodates to infinite which produces a sharp thin line in the same orientation as the line of the laser beam that enters the eye. However, a smaller value of $\alpha$ is produced (for distances in the range of 10 cm – 15 cm from the cylindrical lens that produces the line) when the eye accommodates to the near point, i.e. to image the position of the cylindrical lens, which produces a blurred spot with a value of $\alpha$ smaller than for accommodation to infinity at the same exposure position.

2.1 The most hazardous position

In order to identify and understand the dependencies of the MHP, additionally to the angular subtense of the apparent source as function of position in the beam, the power $P$ that passes through a 7 mm aperture for the different positions in the beam need to be considered. For beam diameters sufficiently smaller than 7 mm, practically the total power passes through the aperture. Therefore, around a certain range around the beam waist, moving the aperture (and the eye) away from the beam waist reduces $P$ relatively little, however, $\alpha$ (being the angle that is subtended by the beam waist) decreases linearly with greater distances of the eye from the beam waist, decreasing the limit. Thus the overall level of hazard, the ratio of $P$ over the limit (or in relative terms, $P/\alpha$), increases with increasing distance to the beam waist as $P$ more or less remains unchanged but $\alpha$ decreases with distance. This holds up to the position where the beam diameter approaches the diameter of the aperture, which constitutes the MHP, as for distances beyond that point, the decrease of the power that enters the eye outweighs the decrease of $\alpha$.

If a (hypothetical) beam would have sharp edges (i.e. a top-hat profile) then in the far-field, the beam could be well approximated by a cone with opening angle $\theta_0$ and the peak of the cone located at the beam waist. The MHP would be simply the furthest distance from the beam waist where the full beam passes through the 7 mm aperture, i.e. the MHP would be where the beam diameter equals 7 mm. For further distances, the power that passes through the aperture would be reduced proportional to the square of the distance increment, while the angular subtense of the beam diameter (and thus $\alpha$) is reduced only linearly. For Gaussian beam profiles, where there is no sharp edge, the model (which can also be derived analytically) shows that the MHP is where the beam radius in 2nd moment terms is somewhat larger than the 7 mm aperture, namely 8.8 mm so that 72 % of the total power passes through the 7 mm pupil. When
compared to above figures and to figure 9, we have just described the ‘central’ region in terms of waist diameter and divergence, which is denominated with ‘Region 2’. However, there is also a region of small beam waist diameters and/or small divergence values, where the minimal retinal spot size at the most hazardous position (and therefore for any exposure position) remains below $\alpha_{\min}$. This region is marked as ‘Region 1’, which can be best discussed in detail when subdivided into three regions as discussed further below.

![Diagram](image)

**Fig. 9:** A number of distinctive regions of ‘beams’ in terms of beam divergence and beam waist diameter can be distinguished, each of which can be understood on the basis of the dependence of the ratio of the power that passes through a 7 mm pupil and the angular subtense of the apparent source $\alpha$ on the position in the beam.

For divergence $\theta_\sigma > 0.9$ rad and waist diameters $d_\sigma > 0.21$ mm, Region 3, covers beams where the MHP is at 10 cm from the beam waist, the near point of the eye, and the beam diameter at 10 cm is larger than 8.8 mm, depending on the divergence. If the eye could accommodate to closer positions than 10 cm, region 2 would be continued also for divergence values above 0.9 mrad, however, with a near point of 10 cm, the retinal spot increases for smaller distances to the beam waist as 10 cm and this makes the exposure less hazardous, even though more power would pass through the 7 mm pupil at distances closer to the beam waist. The angular subtense of the apparent source is given again by the angular subtense of the beam waist, i.e. the eye images the beam waist to produce the smallest retinal spot (for beams with such high irradiances, the distance of 10 cm from the beam waist is in the far field).

For divergence values larger than $1.5 \cdot \sqrt{2} = 2.1$ mrad and beam waists larger than 9 mm, Region 4 covers the case where the worst case position is the beam waist and $\alpha$ is equal to the divergence (corrected for beam diameter definition) $\alpha = \theta_\sigma / \sqrt{2}$, as the minimal retinal spot is achieved by a relaxed eye that ‘images’ infinity. Therefore $\alpha$ does not depend on the beam waist diameter, but the power that passes through the 7 mm aperture located at the beam waist obviously does.
The three regions 2, 3 and 4 are ones where $\alpha$ at the most hazardous position is larger than the minimal angle of 1.5 mrad and thus the correction factor $C_\alpha$ is larger than 1. Region 1 is defined by a retinal spot size which is less than 1.5 mrad, i.e. $\alpha = \alpha_{\text{min}}$ and can be subdivided into three subregions, depending on which other region where $\alpha > \alpha_{\text{min}}$ they border. In Region 1_2, the location $z$ in the beam where $\alpha = \alpha_{\text{min}}$ (the angular subtense of the waist diameter equals $1.5 \cdot \sqrt{2} = 2.1$ mrad) is closer to the beam waist than the position where beam diameter equals 8.8 mm. Thus at the position where $\alpha = \alpha_{\text{min}}$, the power that passes through 7 mm is higher than 72 % and is actually 100 % for most cases (the top plateau in the 3D power figure 3). Thus the MHP can be approximated well by the formula $z_{\text{haz}} = d_0 / 2.1$ mrad where $zhaz$ is in metres while $d_0$ is in mm. However, for cases where the beam diameter at this MHP is small enough, distances beyond the MHP have the same relative thermal hazard level, as $\alpha$ remains at $\alpha_{\text{min}}$ and the power that passes through the 7 mm pupil is still practically 100 %.

For beams in Region 1_4, the beam divergence is so small that even the relaxed eye (which usually produces the largest retinal spot when the eye is on the diverging side of the beam waist) produces a retinal spot which is below the minimal value of 1.5 mrad, i.e. $\alpha = \theta_\sigma / \sqrt{2} < 1.5$ mrad so that $\theta_\sigma < 2.1$ mrad. Since $\alpha$ is 1.5 mrad for all exposure positions, the maximum hazard is achieved by placing the 7 mm aperture in the beam waist. Region 1_3 covers beams with waist diameters less then 0.21 mm, so that at a distance of 10 cm from the waist, the angular subtense of the waist is less than $1.5 \cdot \sqrt{2} = 2.1$ mrad. The power that passes through a 7 mm aperture located at 10 cm from the beam waist depends on the beam divergence.

Sample beams and corresponding formulas for three of the regions are shown in figure 10.

![Fig. 10: Schematic of the most hazardous position for a beam with varying divergence.](image)

The results show that for beams with a beam parameter product $d_0 \cdot \theta_\sigma$ less than 19.5 mm mrad (which can be transformed with the wavelength into respective $M^2$ values), the angular subtense of the apparent source assumes the minimal value of 1.5 mrad. For 532 nm this would for instance translate to an $M^2$ of 29, for 1064 nm to an $M^2$ of 14.4.
2.2 Limitations of the model
The model is based on the variation of the beam diameter along the propagation direction, and the only output information really is the beam diameter (defined according to the 2nd moment technique [4]) and there is no further information on the irradiance profile on the retina (the image plane) nor on the irradiance profile at the cornea. This would not be a problem if the 2nd moment beam diameter would be directly related to the characteristic thermal diameter on which $\alpha$ is based is the same, but this is unfortunately not the case [3]. Also, the 2nd moment diameter definition can be grossly misleading when it is used to characterise the beam diameter at the cornea for non-Gaussian beams. Calculations of power levels that enter the 7 mm pupil can be off by a factor of 20 or more for irradiance distributions that occur for instance in the far field of instable resonators.

Consequently the model can only be used for laser beams with Gaussian beam profile (TEM$_{00}$), because only for that case the irradiance profile in the image plane is known, as the Gaussian profile is conserved, at least for the assumption of perfect optics and no diffraction by apertures. For beam profiles other than a Gaussian, the model can over- but also seriously under-predict the hazard.

3 SUMMARY AND CONCLUSIONS
A general scheme and understanding of the parameter “angular subtense of the apparent source” is discussed.

Following this general approach, the comparison of the emission with the exposure limit (or emission limit) is done at the most hazardous position in the beam, which is found by varying the position of the eye in the beam and for each position, to consider different accommodation states of the eye to determine the angular subtense of the apparent source for each position of the eye in the beam. Currently, measurement distances are specified relative to the location of the apparent source, which can be problematic, as the location and size of the apparent source is often is not obvious – i.e. not ‘apparent’. The general hazard evaluation method as discussed here is independent on the location of the apparent source.

For a laser beam, the location of the apparent source depends on the exposure position in the beam, and therefore it does not make sense to ask for ‘the’ apparent source for a given laser beam without specifying the exposure position within the beam. From this it also follows that the current specification of IEC 60825-1 to specify the distance of the determination of $\alpha$ relative to the position of the apparent source is actually a paradox.

Since $\alpha$ describes the minimal retinal spot size that can be obtained for a given exposure position (or for the case of the general approach, for the MHP), it might be less confusing to refer to the angular subtense of the minimal retinal image rather than the angular subtense of the apparent source.

It is important to note that $\alpha$ is equal to the divergence of the beam only in exceptional cases, namely when the eye is at the position or close to the beam waist (i.e. in the near field) or when the eye has a fixed accommodation to infinity. In other cases, the value of $\alpha$ will be smaller than the far field divergence of the beam.

ACKNOWLEDGEMENTS
The beam propagation computer model was realised by Ulfried Grabner and Sandra Althaus, based on an initiative by Brooke Ward to apply 2nd moment beam diameter definitions and beam propagation in the eye to that problem.

The author gratefully acknowledges fruitful discussions with Brooke Ward on beam propagation modelling and 2nd moment beam diameter definitions as well as with Bernd Eppich from TU Berlin on the limitations thereof.
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